EoS constraints from a model-independent approach

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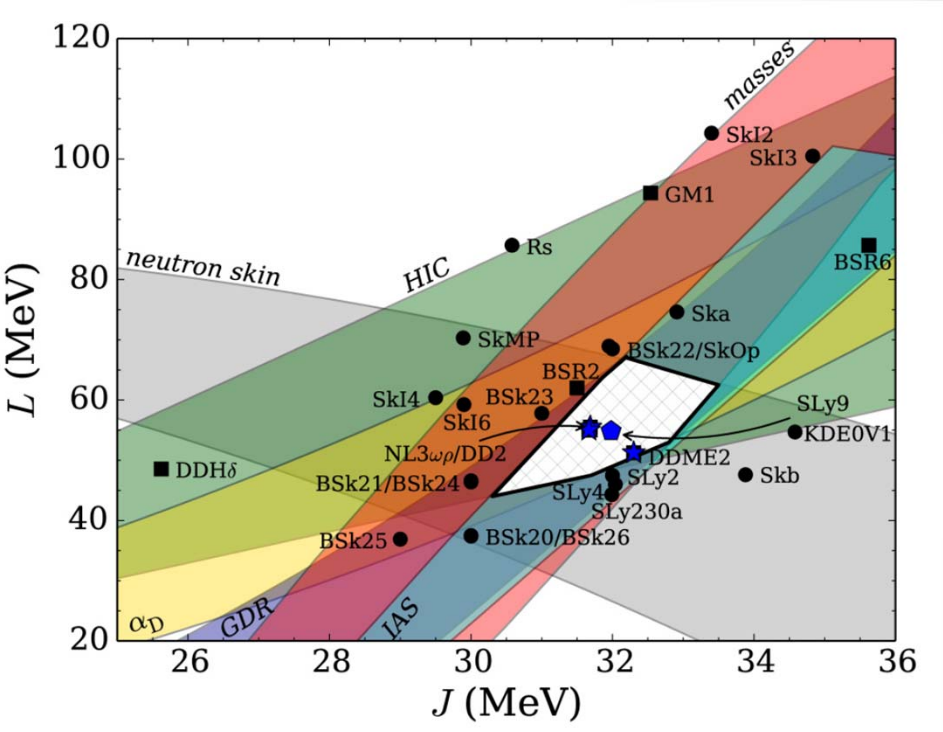
EoS and empirical constraints

- EoS can be characterized by empirical parameters $P_k = \partial^k e/\partial\rho^k$ ex:J,L,…
- DFT models corresponding to different EoS are compared to exp.data
- $P_k \pm \Delta P_k$ determined fitting the model to the data
- Correlations among P_k are typically observed



M.Fortin et al, PRC 94,035804

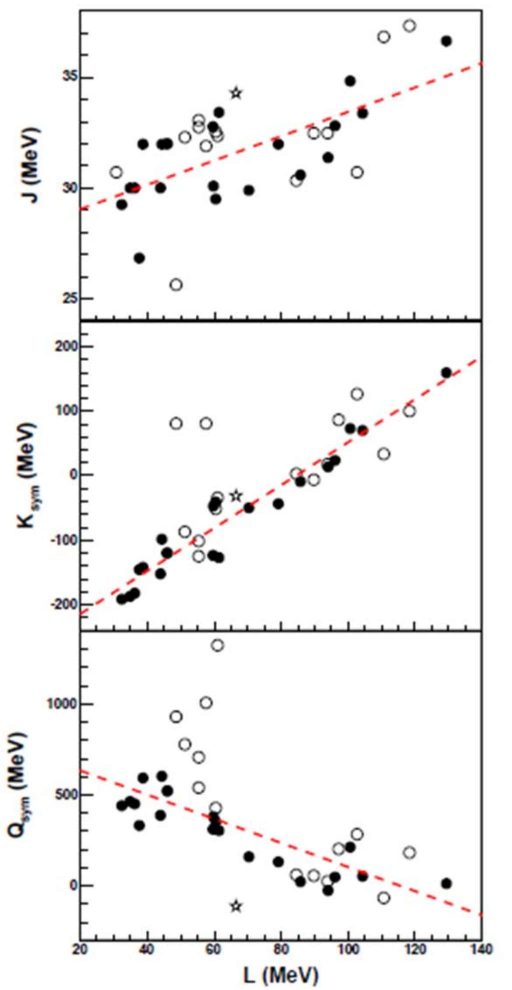
Problems

- Nuclei are not simply droplets of nuclear matter! Energy functionals contain many terms

  => Uncertainty in gradient couplings gets mixed up with uncertainty in P_k

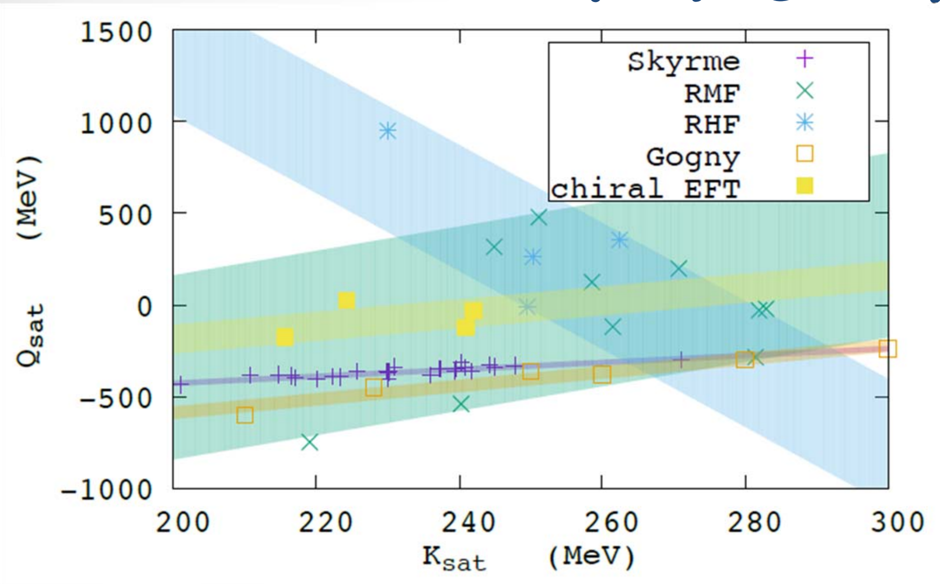
- Pheno functionals contain spurious correlations among emp.parameters

  => results are model dependent



C.Ducoin et al PRC 2011

Problems



- Pheno functionals contain spurious correlations among emp.parameters

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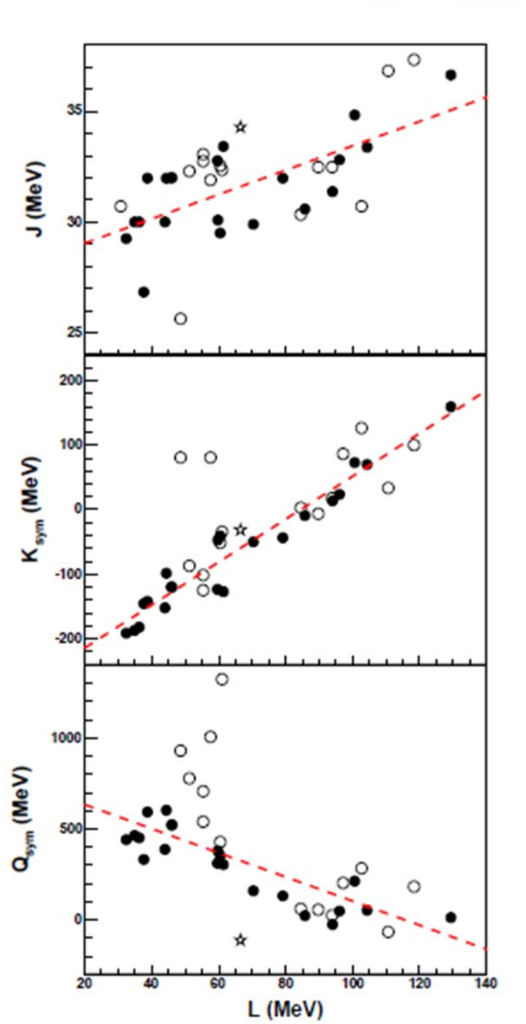


C.Ducoin et al PRC 2011

A model independent approach

- Nuclei are not simply droplets of nuclear matter! Energy functionals contain many terms
  - => Uncertainty in gradient couplings gets mixed up with uncertainty in P_k

A single effective isoscalar gradient term
to be fitted on nuclear masses

$$e(n,\delta) = e_{NM} + C(\nabla n)^2$$

$$\begin{cases} n = n_p + n_n \\ \delta = (n_p - n_n)/n \end{cases}$$

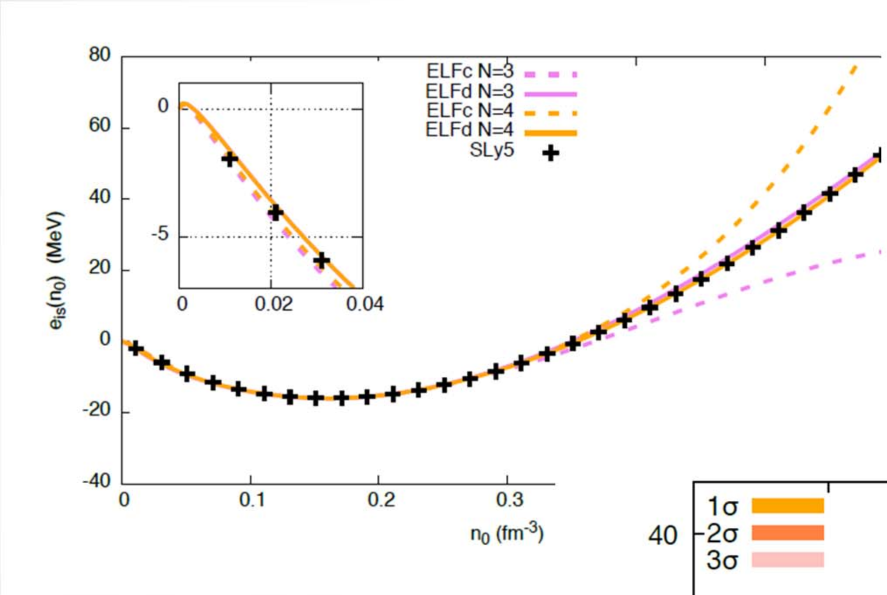
- Pheno functionals contain spurious correlations among emp.parameters
  - => results are model dependent

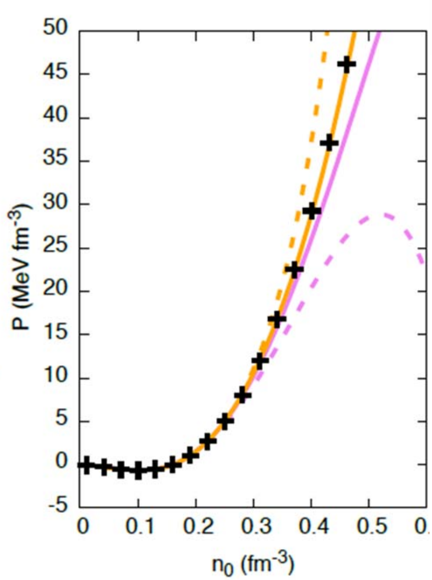
Taylor expansion around n_0

$$e_{NM}(n,\delta) = \sum_k \frac{1}{k!}\left(c_k^{IS} + c_k^{IV}\delta^2\right)\left(\frac{n-n_0}{3n_0}\right)^k$$

Steiner, Lattimer, Brown ApJ722(2010)33

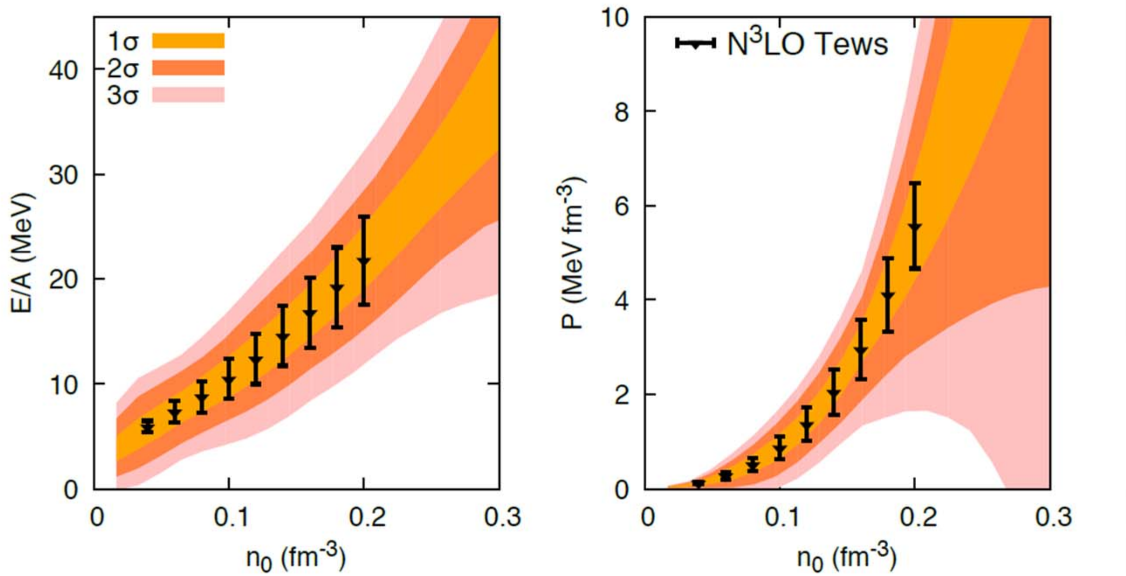
HNM: Quality of the Taylor expansion





Sly5

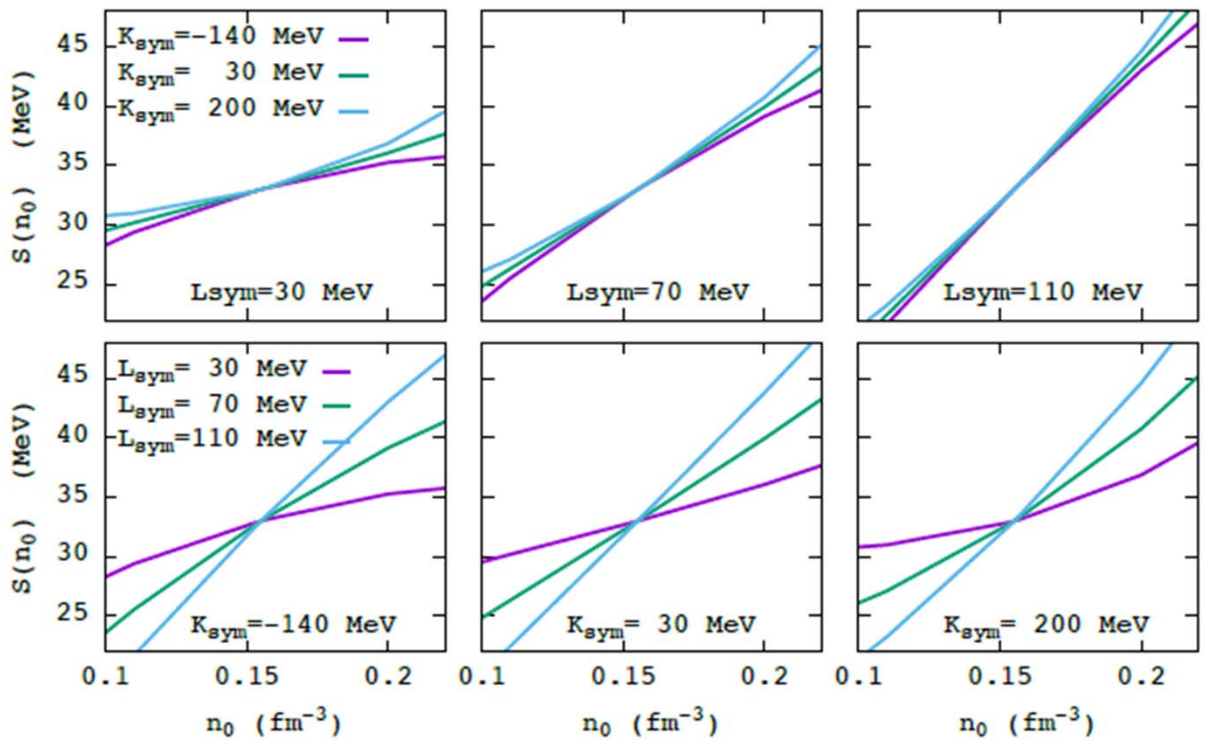
E.Chabanat, P.Bonche, P.Hansel, J.Meyer, R.Schaeffer, NPA627(1997)710



Chiral EFT

I.Tews, T.Kruger,K.Hebeler,A.Schwenk PRL110(2013)032504

Symmetry energy



Present uncertainty on P_k: prior distribution

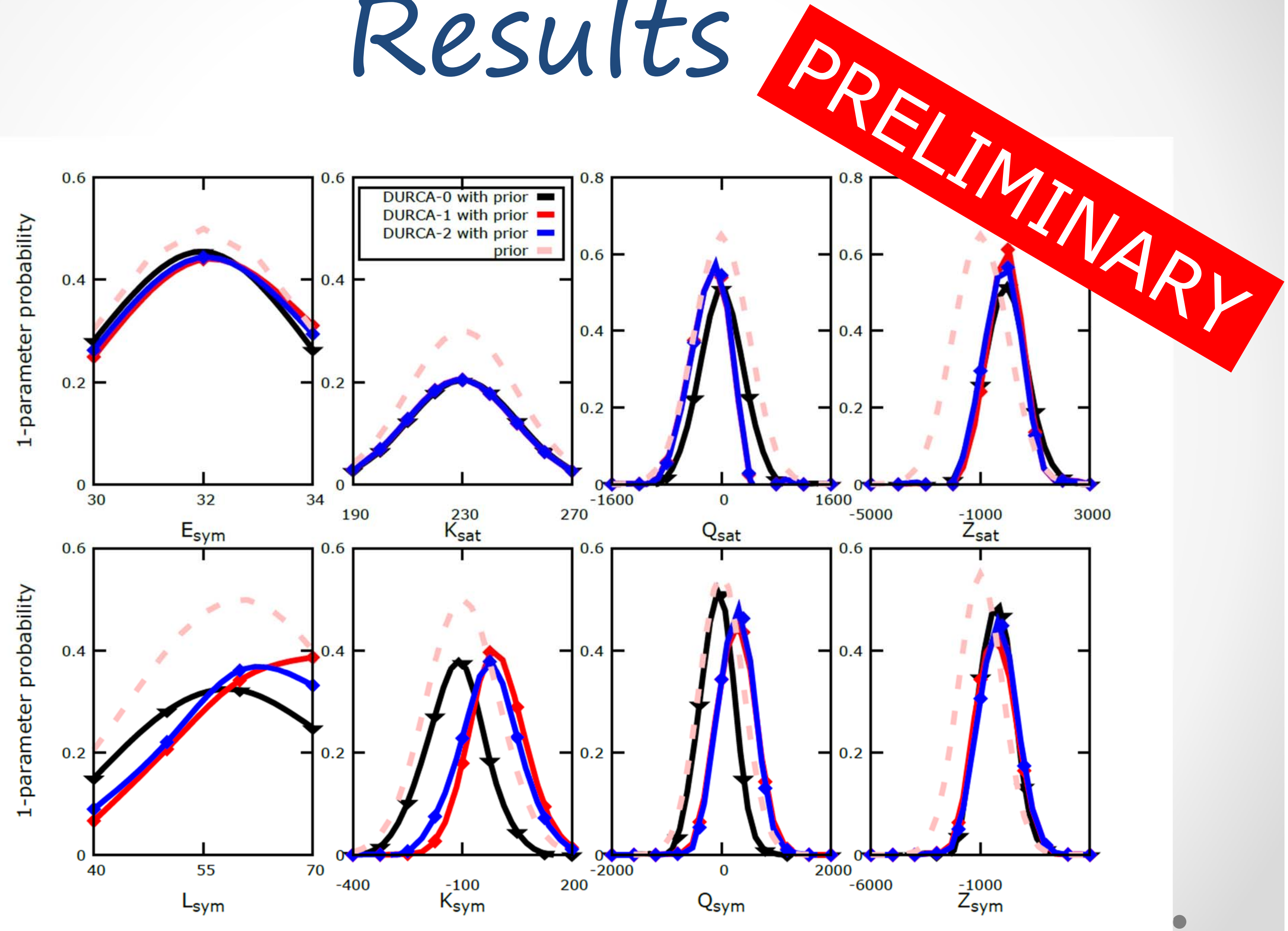
Model		E_{sat} MeV	E_{sym} MeV	n_{sat} fm^{-3}	L_{sym} MeV	K_{sat} MeV	K_{sym} MeV	Q_{sat} MeV	Q_{sym} MeV	Z_{sat} MeV	Z_{sym} MeV	m^*_{sat}/m	$\Delta m^*_{sat}/m$
(N_α)	der. order	0	0	1	1	2	2	3	3	4	4	-	-
						Phenomenological approaches							
Skyrme	Average	-15.88	30.25	0.1595	47.8	234	-130	-357	378	1500	-2219	0.73	0.08
(16)	σ	0.15	1.70	0.0011	16.8	10	66	22	110	169	617	0.10	0.24
Skyrme	Average	-15.87	30.82	0.1596	49.6	237	-132	-349	370	1448	-2175	0.77	0.127
(35)	σ	0.18	1.54	0.0039	21.6	27	89	89	188	510	1069	0.14	0.310
RMF	Average	-16.24	35.11	0.1494	90.2	268	-5	-2	271	5058	-3672	0.67	-0.09
(11)	σ	0.06	2.63	0.0025	29.6	34	88	393	357	2294	1582	0.02	0.03
RHF	Average	-15.97	33.97	0.1540	90.0	248	128	389	523	5269	-9956	0.74	-0.03
(4)	σ	0.08	1.37	0.0035	11.1	12	51	350	237	838	4156	0.03	0.01
						Ab-initio approaches							
APR	Average	-16.0	33.12	0.16	50.0	270	-199	-665	923	337	-2053	1.0	0.0
(1)	σ	-†	0.30	-†	1.3	2	13	30	67	94	125	-†	-†
χ-EFT	Average	-15.16	32.01	0.171	48.1	214	-172	-139	-164	1306	-2317	-	-
Drischler 2016	σ_{tot}	1.24	2.09	0.016	3.6	22	40	104	234	214	379	-	-
(7)	Min	-16.92	28.53	0.140	43.9	182	-224	-310	-640	901	-2961	-	-
	Max	-13.23	34.57	0.190	53.5	242	-108	24	96	1537	-1750	-	-

P_α	E_{sat} MeV	E_{sym} MeV	n_{sat} fm^{-3}	L_{sym} MeV	K_{sat} MeV	K_{sym} MeV	Q_{sat} MeV	Q_{sym} MeV	Z_{sat} MeV	Z_{sym} MeV	m^*_{sat}/m	$\Delta m^*_{sat}/m$
$\langle P_\alpha \rangle$	-15.8	32	0.155	60	230	-100	0	0	-1000	-1000	0.75	0.1
σ_{P_α}	$\pm$0.3	$\pm$2	$\pm$0.005	$\pm$15	$\pm$20	$\pm$100	$\pm$400	$\pm$400	$\pm$1000	$\pm$1000	$\pm$0.1	$\pm$0.1

HNM: Constraints from neutron star physics

- Causality: $0 < v_s < c$
- NS stability: $\nabla p > 0$ for $n > n_0$ in β-equilibrium
- $E_{sym} > 0$
- $M_{max} > 2M_o$
- Limit on DURCA:
  - No DURCA up to $2M_o$ – DURCA0
  - DURCA only for $M > 1.8M_o$ – DURCA1
  - DURCA only for $M > 1.6M_o$ – DURCA2

Results



PRELIMINARY
DURCA-0 with prior
DURCA-1 with prior
DURCA-2 with prior
prior
1-parameter probability
E_{sym}
K_{sat}
Q_{sat}
Z_{sat}
L_{sym}
K_{sym}
Q_{sym}
Z_{sym}

PRELIMINARY

	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
Z_{sym}	0.0	0.1	-0.4	-0.0	-0.1	-0.2	[illegible]	1.0
Q_{sym}	-0.0	-0.2	0.1	-0.0	-0.1	-0.2	1.0	[illegible]
K_{sym}	0.0	-0.0	0.1	-0.0	-0.1	1.0	-0.2	-0.2
L_{sym}	0.0	0.0	0.0	-0.0	1.0	-0.1	-0.1	-0.1
E_{sym}	-0.0	-0.0	0.0	1.0	-0.0	-0.0	-0.0	-0.0
Z_{sat}	0.0	-0.1	1.0	0.0	0.0	0.1	0.1	-0.4
Q_{sat}	-0.0	1.0	-0.1	-0.0	0.0	-0.0	-0.2	0.1
K_{sat}	1.0	-0.0	0.0	-0.0	0.0	0.0	-0.0	0.0

Finite nuclei

- Nuclei are not simply droplets of nuclear matter! Energy functionals contain many terms

  => Uncertainty in gradient couplings gets mixed up with uncertainty in P_k

A single effective isoscalar gradient term
to be fitted on nuclear masses

$$e(n,\delta) = e_{NM} + C(\nabla n)^2$$

Observables from $\hbar^2$-ETF with parametrized density profiles

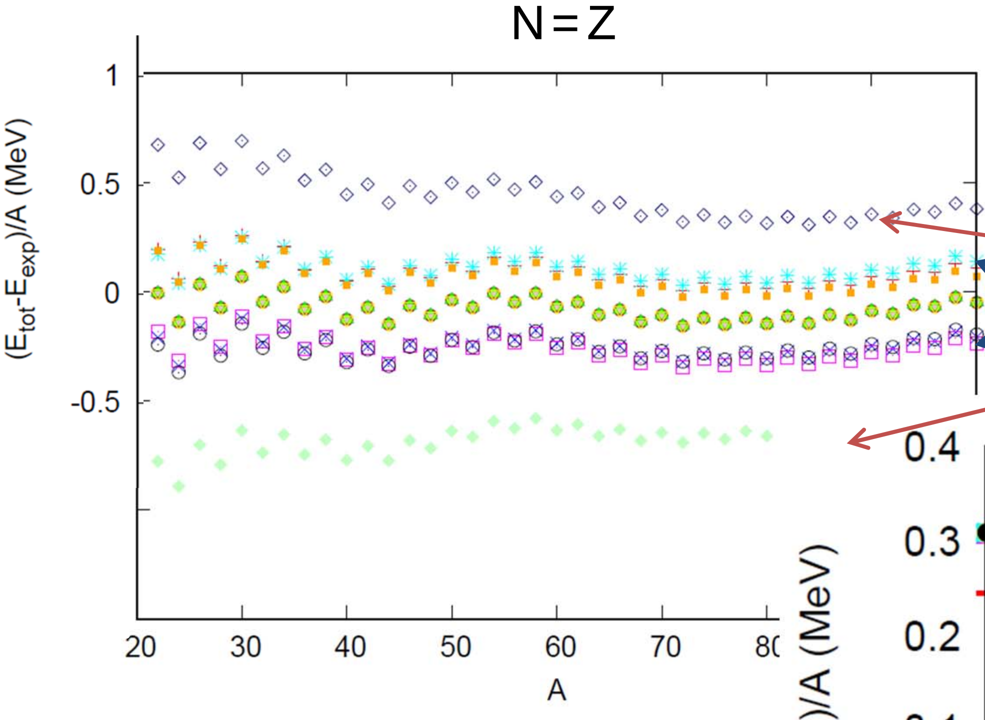
$$n_q(r) = \frac{n_{0q}}{1 + e^{\frac{r-R_q}{a_q}}}$$

Analytical integration of the Fermi integrals

F.Aymard et al., J.Phys.G43,045105(2016)

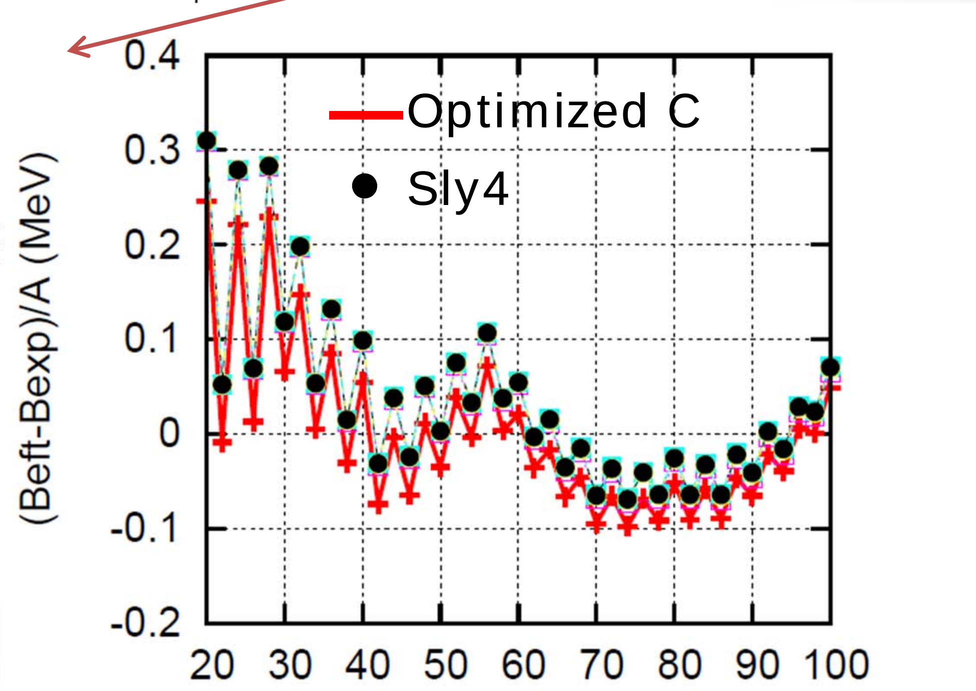
Calibrating the gradient term

$$e(n,\delta\,) = e_{bulk} + C(\nabla n)^2$$



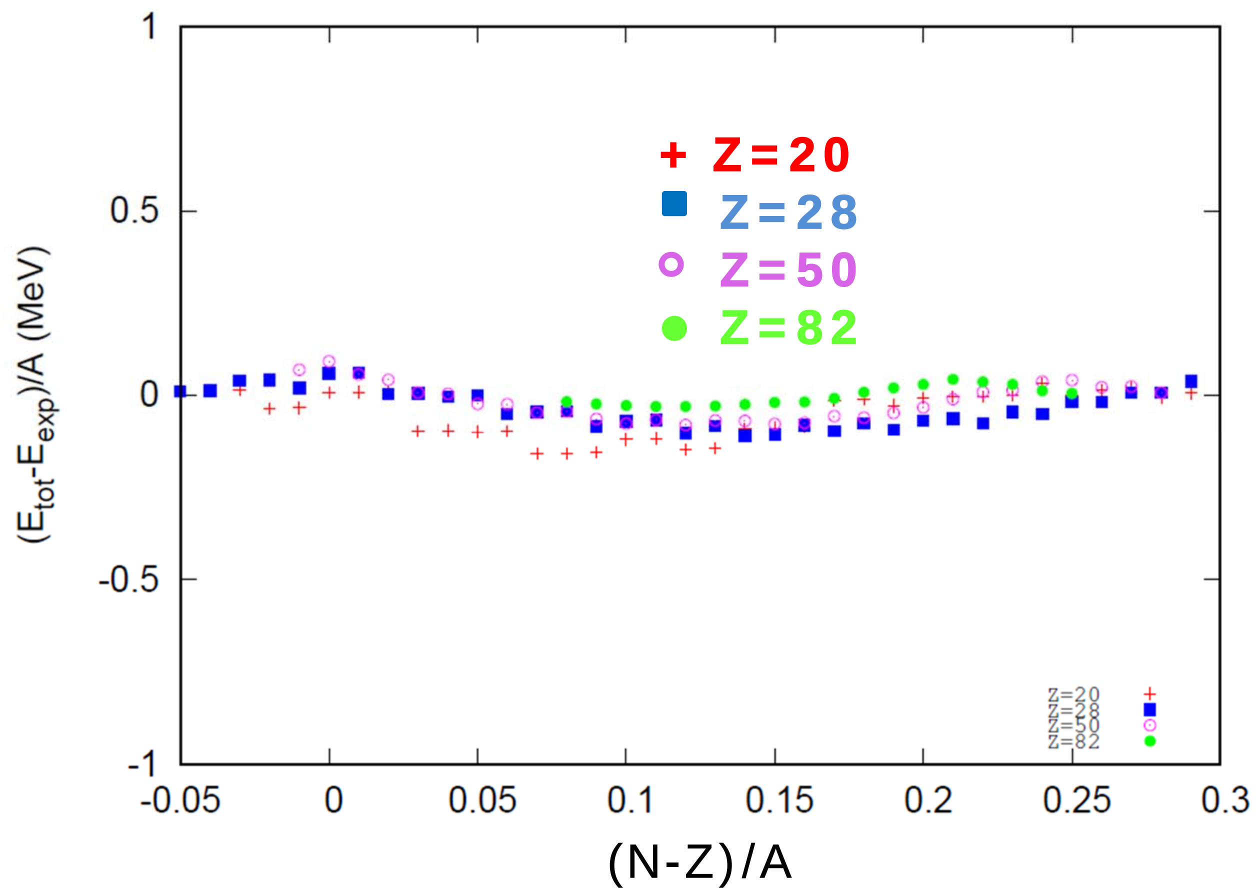
$P_k \pm \Delta P_k$

$\mathrm{C} \pm 20$



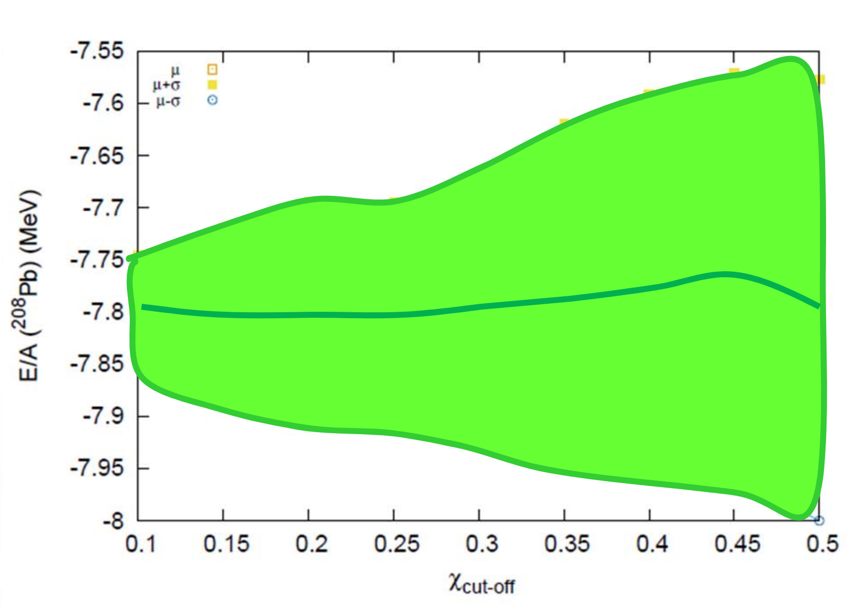
- Residual deviations are due to the semi-classical approximation

Semi-magic isotopic chains

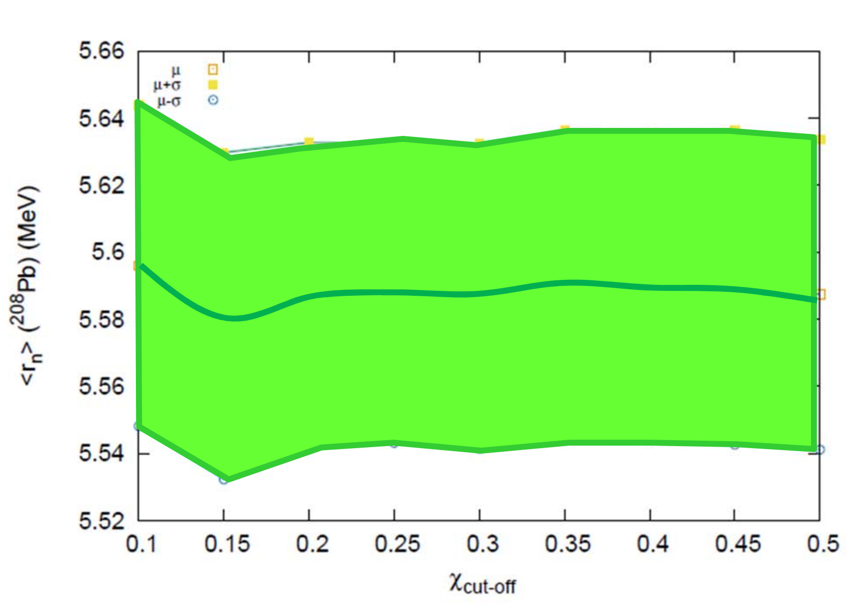


Exploring the parameter space

$$\chi = \frac{1}{N_{tot}} \left[\sum_{I=0} \frac{|\Delta E|}{A} + \sum_{Z=20} \frac{|\Delta E|}{A} + \sum_{Z=28} \frac{|\Delta E|}{A} + \sum_{Z=50} \frac{|\Delta E|}{A} + \sum_{Z=82} \frac{|\Delta E|}{A} \right] ,$$



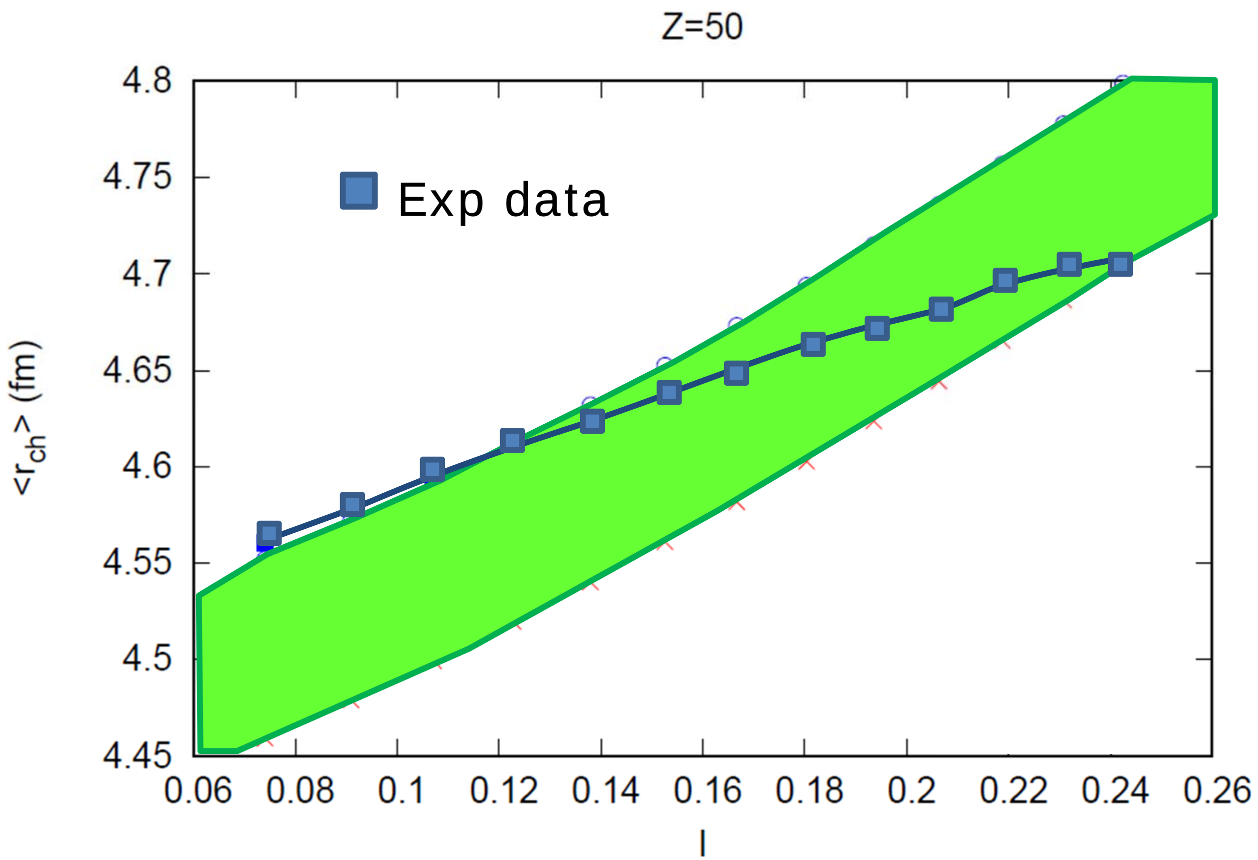
Average binding-energy deviation



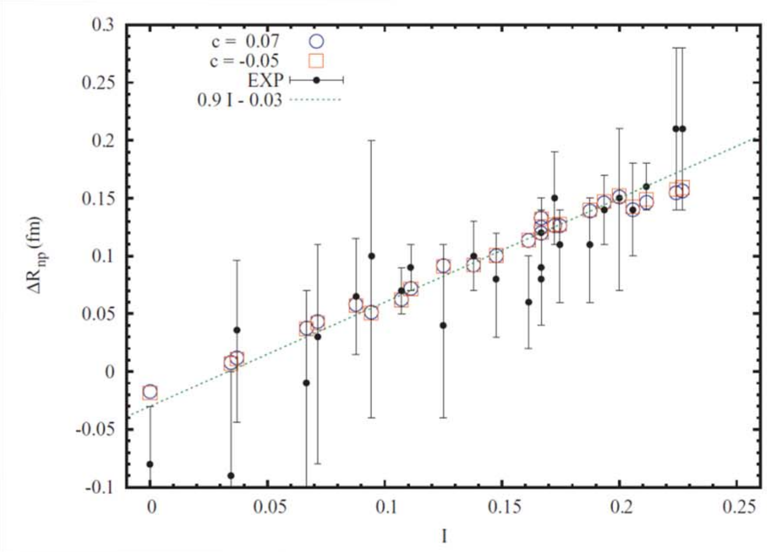
Average binding-energy deviation

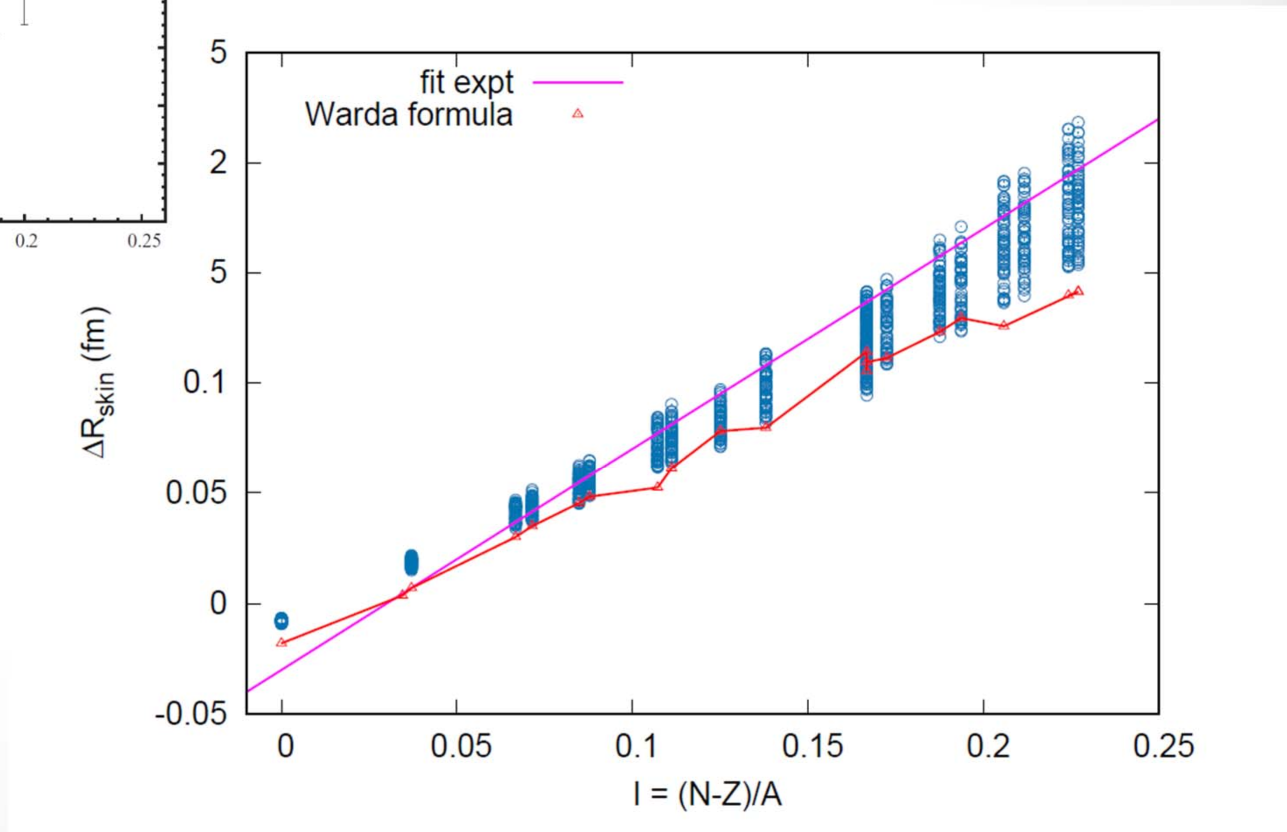
Results: radii

- After filter; χ_{cutoff}=0.2 MeV

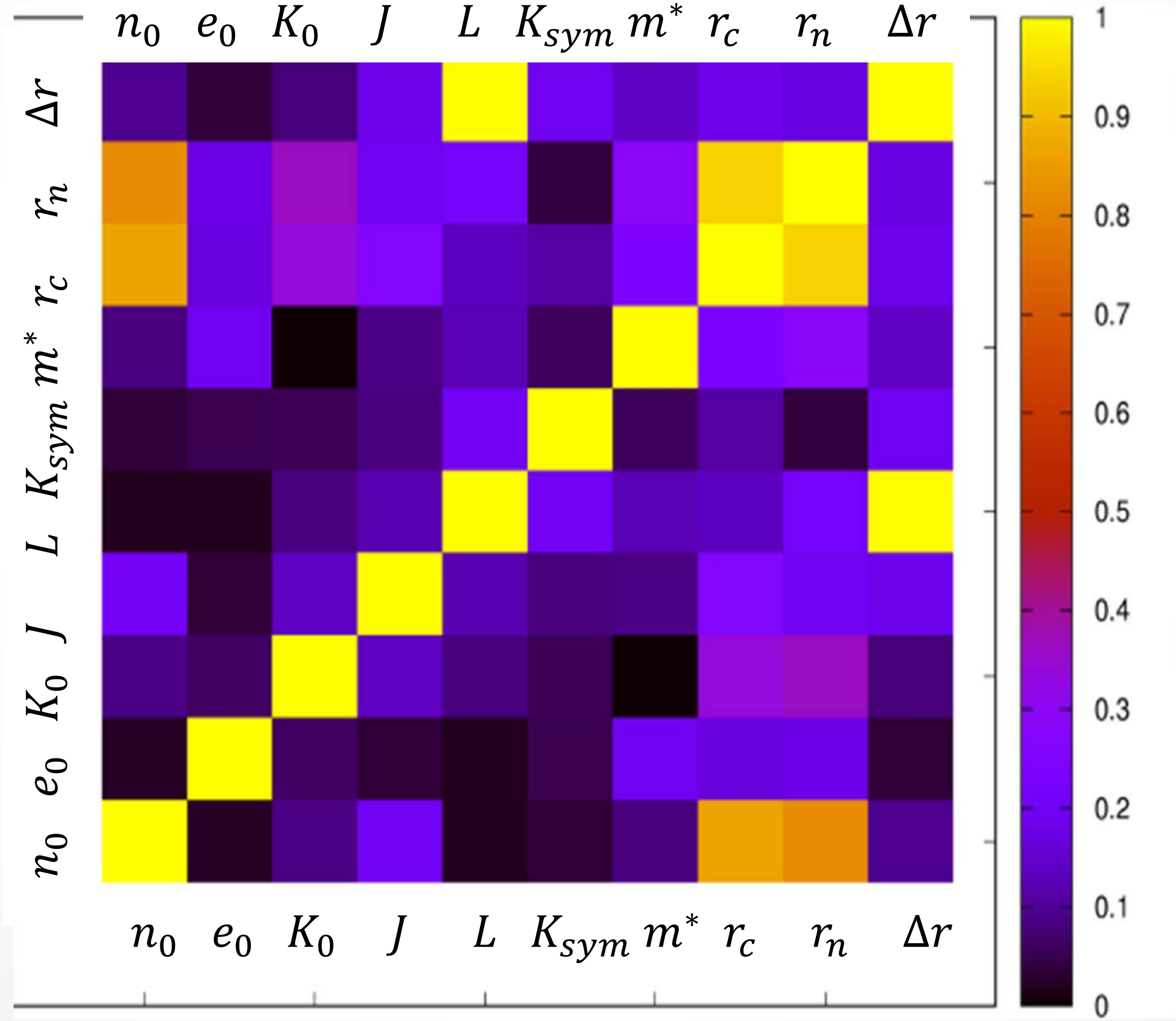


Results: n-skin





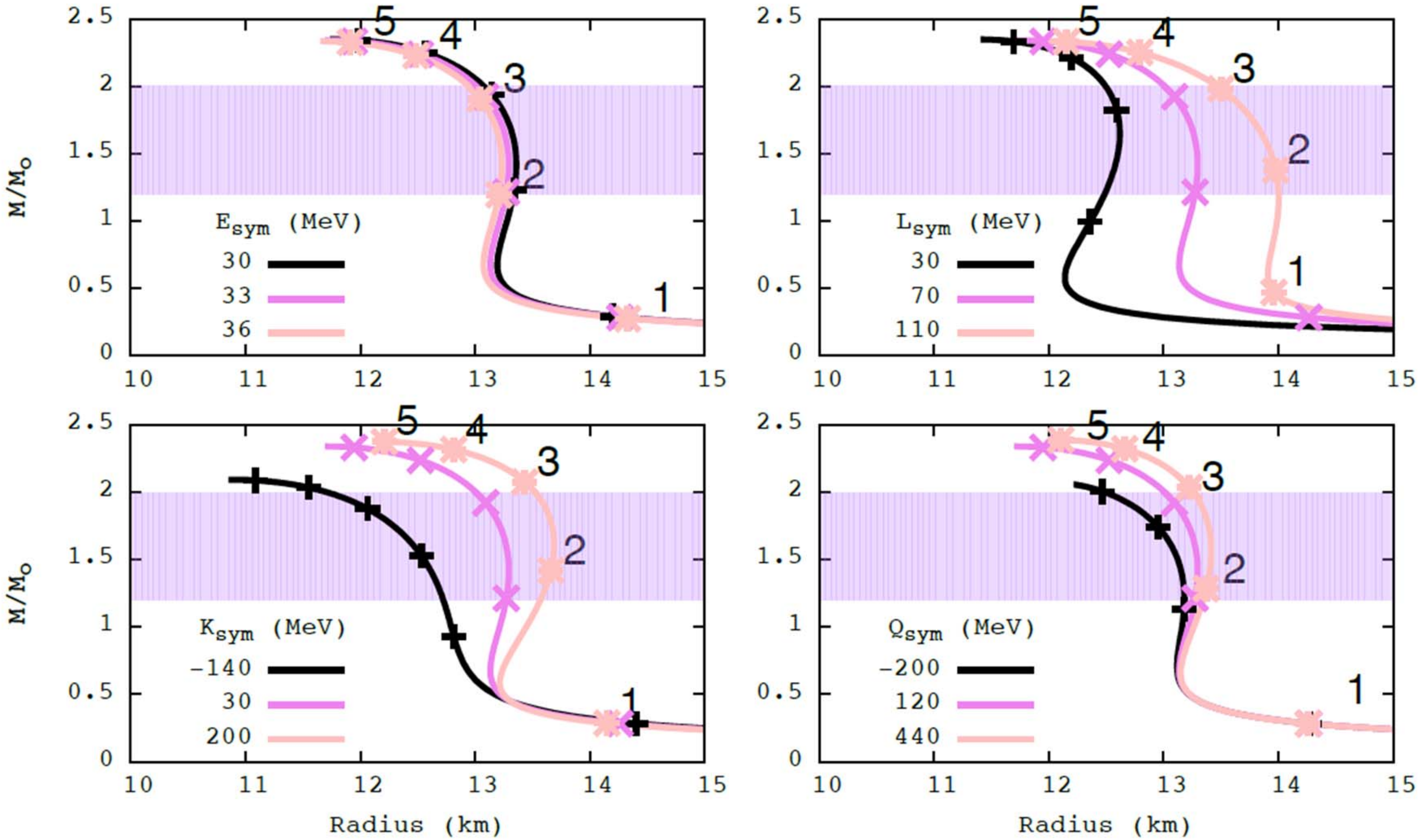
Results: correlations



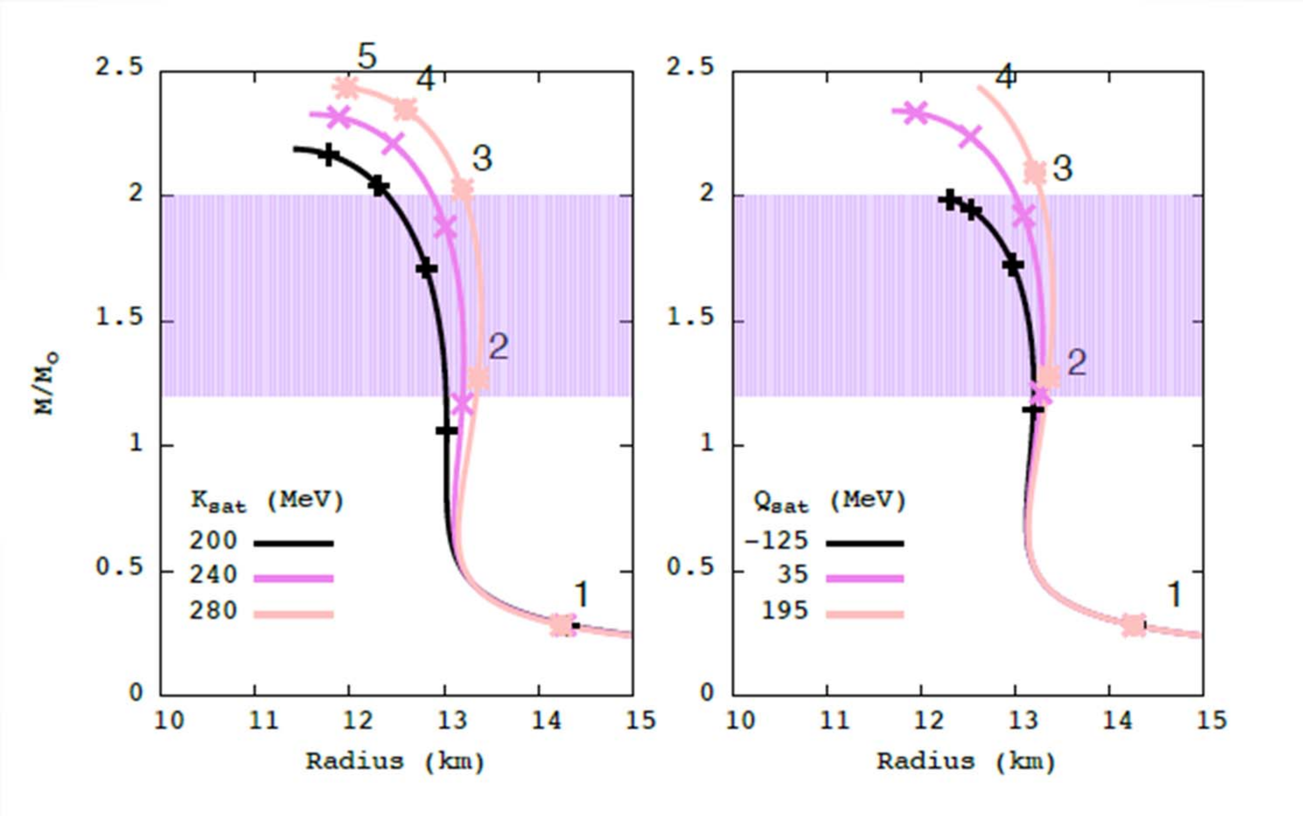
Conclusions

- Constraints on EoS empirical parameters need both NS physics and laboratory experiments
- We propose an empirical EoS avoiding spurious constraints from the energy density functional form
- Finite nuclei observables from ETF with a single gradient term fixed from nuclear mass
- Bayesian determination of parameters with flat or gaussian prior

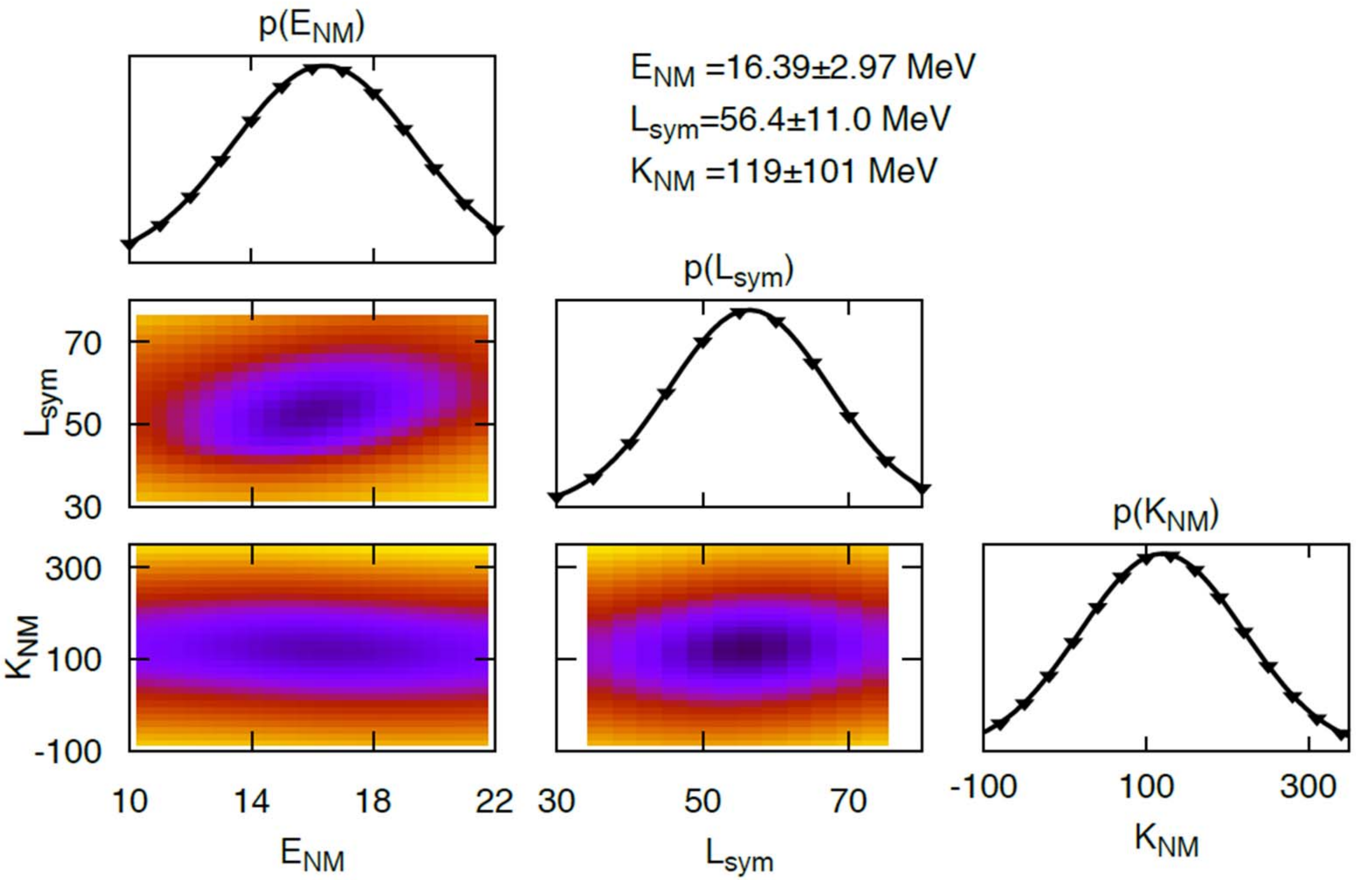
- **Third order derivatives still largely unconstrained**
- **SKIN CORRELATED TO L**
- **ALMOST NO CORRELATION AMONG EMP.PARAMETERS**



M/M☉
2.5
2
1.5
1
0.5
0
10
11
12
13
14
15
Radius (km)
E_sym (MeV)
30
33
36
L_sym (MeV)
30
70
110
K_sym (MeV)
-140
30
200
Q_sym (MeV)
-200
120
440
1
2
3
4
5



2.5
2
1.5
1
0.5
0
10 11 12 13 14 15
Radius (km)
M/M_O
5
4
3
2
1
K_sat (MeV)
200
240
280
2.5
2
1.5
1
0.5
0
10 11 12 13 14 15
Radius (km)
4
3
2
1
Q_sat (MeV)
-125
35
195


p(E_NM)
E_NM =16.39±2.97 MeV
L_sym=56.4±11.0 MeV
K_NM =119±101 MeV
p(L_sym)
p(K_NM)
L_sym
70
50
30
K_NM
300
100
-100
10
14
18
22
30
50
70
-100
100
300
E_NM
L_sym
K_NM